

**COMMENTS ON "FLUX-GRADIENT RELATIONSHIP, SELF-CORRELATION
AND INTERMITTENCY IN THE STABLE BOUNDARY LAYER"**

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A comment note submitted to the "Quarterly Journal of Royal Meteorological Society"

August 5, 2005

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Klipp and Mahrt (2004) demonstrate, based on data obtained during CASES-99, that the similarity function for the wind velocity gradient, $\Phi_m = \kappa z/u_* dU/dz$, departs from the linear form $1 + 4.7 z/L$ (e.g., Businger et al., 1973) in very stable conditions ($z/L > 1$), where $u_* = \tau_o^{1/2}$, $T_* = -H_o/u_*$, κ is the von Karman constant, $L = u_*^2/[\kappa \beta T_*]$ is the Monin-Obukhov length, τ_o and H_o are the surface fluxes of the momentum and temperature. The authors argue convincingly that the phenomenon is associated with self-correlation (i.e., a significant correlation coefficient between considered variables), since both expressions, Φ_m and z/L , contain a common divisor u_* . Large random errors for very stable conditions increase the self-correlation, and consequently produce false results. They conclude that consequently a relationship between Φ_m and z/L in very stable conditions might never be empirically established with satisfactory confidence. The purpose of this note is to show that the reported departure can also be related to a deficiency of Monin-Obukhov's flux-based scaling in very stable conditions ($L < 1$ m), and the presence of another similarity regime, characterized by a gradient-based scaling.

The Monin-Obukhov similarity yields the following predictions for mean values of the absolute temperature and wind velocity in stable (z-less) conditions:

$$\begin{aligned} dT/dz &\sim T_*/L_* \sim \beta H_o^2/\tau_o^2 \sim const \\ dU/dz &\sim u_*/L_* \sim \beta H_o/\tau_o \sim const \end{aligned} \tag{1}$$

The above expressions imply that the gradient Richardson number in this case is sub-critical, $Ri = (\beta dT/dz)/(dU/dz)^2 \sim \beta T_* L_*/u_*^2 \sim 1$.

In the strongly stable case, characterized by weak winds, clear skies, strong

radiative cooling near the surface, and intermittent turbulence (e.g., Businger et al., 1973, Van de Wiel et al., 2003; Mahrt, 2003), $H_0 \sim 0$, $\tau_0 \sim 0$, and the ratio H_0/τ_0 in (1) might not be well defined (as seen from the CASES-99 data plotted by e.g., Steeneveld et al., 2004). As a result, the Monin-Obukhov similarity predictions for dT/dz and dU/dz cannot be accurately determined. Moreover, because $dT/dz \rightarrow \infty$ (strong radiative cooling near the earth's surface) and $dU/dz \rightarrow 0$ (calm conditions near the surface), the Richardson numbers are expected to be overcritical. This implies that the set of the governing parameters of Monin-Obukhov similarity theory: τ_0 , H_0 , and β , is improper in the very stable regime.

Following the above conclusion, one might choose an alternate set of governing parameters in the strongly stable case: the buoyancy parameter $\beta = g/T_\infty$, the gradients dT/dz , dU/dz , and the vertical velocity variance σ_w^2 (note that fluxes are excluded from this list). Based on Buckingham's Π -theorem (e.g., Sorbjan, 1995), the following three local (z -dependent) scales can be obtained:

$$\begin{aligned}
 u_n(z) &= \sigma_w \\
 L_n &= -\frac{\sigma_w}{\sqrt{\beta dT/dz}} \\
 T_n(z) &= L_n dT/dz
 \end{aligned} \tag{2}$$

The Π -theorem implies that any statistical moment, scaled in terms of (2), should be a function of the gradient Richardson number $Ri = \frac{\beta dT/dz}{(dU/dz)^2}$. For instance, for the scalar

fluxes, we will have:

$$-\frac{\overline{w'\theta'}}{u_n T_n} = f_H(Ri) \quad (3)$$

$$-\frac{\overline{u'w'}}{u_n^2} = f_M(Ri)$$

which is equivalent to:

$$\overline{w'\theta'} = -\frac{\sigma_w^2 f_H(Ri)}{\sqrt{\beta} dT/dz} dT/dz \quad (4)$$

$$\overline{u'w'} = -\sigma_w^2 f_M(Ri)$$

where f_H and f_M are empirical functions. The variables in (3) are not self-correlated. The term on the right hand side of the first equation in (4), before the temperature gradient, can be recognized as the eddy diffusivity K_H . When $\sigma_w \rightarrow 0$, then $\overline{w'\theta'}$, $\overline{u'w'}$ $\rightarrow 0$, as expected in a very stable, intermittent case. Equations (4) imply that both fluxes are dependent on height.

The second expression in (4) can also be obtained in the flux-gradient form, by assuming that in addition to the vertical velocity scale u_n , the horizontal velocity scale can be defined as $u_h = L_n dU/dz$. Consequently:

$$\overline{u'w'} = -u_h u_n f(Ri) = -\frac{\sigma_w^2 f(Ri)}{\sqrt{\beta} dT/dz} dU/dz = -\frac{\sigma_w^2 f(Ri)}{Ri^{1/2}} \quad (5)$$

which implies that $f_M(Ri) = f(Ri)/Ri^{1/2}$. The term on the right hand side of (5), before the velocity gradient, can be identified as the eddy diffusivity K_m . One might argue that $f(Ri)$

→ $Ri^{1/2}$, when $Ri \rightarrow 0$ (neutral limit), which yields: $K_m \sim u_* z$ and $-\overline{u'w'} \sim u_*^2$.

In order to compare predictions of both considered scaling approaches, from (4) and (5) it can be formally obtained (for non-zero fluxes):

$$\begin{aligned} dU/dz &\propto \frac{dT/dz}{\overline{w'\theta'}} \frac{\overline{u'w'}}{f_H(Ri)} \\ dT/dz &\propto \frac{\sqrt{\beta dT/dz}}{\overline{u'w'}} \frac{\overline{w'\theta'}}{f_H(Ri)} \end{aligned} \quad (5)$$

It should be stressed that the above expressions should not be understood as new similarity predictions for gradients, as such gradients remain external parameters in (2).

Ignoring the dependence of the fluxes on height, (5) can be rewritten in the form:

$$\begin{aligned} \Phi_m &= \frac{\kappa z}{u_*} \frac{dU}{dz} \propto \frac{z}{L} s^2 f_1(Ri) \\ \Phi_h &= \frac{\kappa z}{T_*} \frac{dT}{dz} \propto \frac{z}{L} s f_2(Ri) \end{aligned} \quad (6)$$

where $s = \frac{Nu_*}{\beta T_*}$ is a dimensionless parameter, while $N = \sqrt{\beta dT/dz}$ is the Brunt-Väisälä frequency. The parameter s can be reduced from the above equations, based on the relationship between Ri and s . Such a relationship can be obtained from (6) in the form: $s^3 = Ri^{-1} f_2(Ri)/f_1^2(Ri)$. The expressions (6) indicate that the form of Φ_m and Φ_h in very stable conditions will be disturbed by terms dependent on the Richardson number Ri (and consequently on z/L).

Concluding, the departure from similarity predictions, detected by the authors in

their Figure 3, could be related not only to the self-correlation, but also to the presence of another similarity regime (characterized by the gradient-based scaling) in very stable conditions ($z/L > 1$, and $L < 1$ m), for which the Monin-Obukhov scaling and predictions are not valid.

Acknowledgments: The research has been supported by the National Science Foundation grant No. ATM-0400590.

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